Numerical scheme for modelling of nonlinear pulse propagation in coupled microring resonators

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1. Introduction

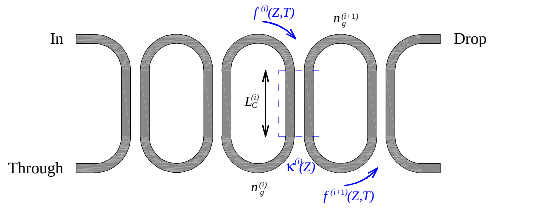
Numerical techniques for simulation of nonlinear light propagation are of fundamental importance in the analysis and design of new functional devices. Considering coupled microring structures, theoretical studies of their nonlinear properties are often based on a nonlinear variant of the transfer matrix method (TMM) [1,2,3]. However, the method requires a solution of a nonlinear matrix problem, which may not always quickly converge nor be unique, and provides results in the frequency domain only. The latter can limit the usefulness of the technique. For example, the TMM cannot describe dynamics of optical bistability, a phenomenon, which is expected to play an important role in data processing applications [4]. Coupled resonant structures may also exhibit further interesting effects, such as generation of optical pulses from continuous wave input (self-pulsing) [5], which cannot be simulated in the frequency domain at all. These constraints can be overcome by using the finite-difference time-domain (FD-TD) method. However, for an exact description of resonant structures it is necessary to use very high spatial resolutions, resulting in time-consuming calculation, and/or apply advanced algorithms with correction of the phase velocity error [6,5].

Recently, we presented a simple numerical technique that avoids some of the mentioned problems [7]. Under the slowly-varying envelope approximation, propagation of optical pulses in coupled microring systems is described by a system of coupled partial differential equations. These equations are solved by an explicit finite-difference scheme based on upwind differencing. We will denote the technique as CE (coupled equations).

Here, we present more detailed analysis of CE method. We study accuracy of various scheme alternatives and present stability criterions. Then the CE technique is used for a simulation of more complex structures, a Kerr-nonlinear optical channel dropping filters, which include 1 and 3 microrings. The structures exhibit optical bistability and self-pulsing and were chosen with the aim to demonstrate typical circumstances in which the CE method may be useful.

2. CE method

Propagation of optical pulses in coupled microring resonators



Equations for slowly varying envelopes

$$\frac{\partial f^{(i)}}{\partial Z} + n_g^{(i)}\frac{\partial f^{(i)}}{\partial T} = i\kappa^{(i)}(Z) f^{(i+1)} + i n_2^{(i)} \left|f^{(i)}\right|^2 f^{(i)}$$

$$\frac{\partial f^{(i+1)}}{\partial Z} + n_g^{(i+1)}\frac{\partial f^{(i+1)}}{\partial T} = i\kappa^{(i)}(Z) f^{(i)} + i n_2^{(i+1)} \left|f^{(i+1)}\right|^2 f^{(i+1)}$$

normalized spatial and time variables $Z = z\frac{2\pi}{\lambda}$ $T = t\frac{2\pi c}{\lambda}$

Kerr nonlinearity

if $\kappa(Z) = \text{const}$ $s^{(i)} = \sin\left(\kappa^{(i)} L_C\right)$

imaginary part of the net coupling coefficient

normalized coupling coefficient

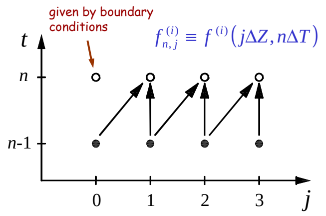
FD explicit scheme (upwind differencing)

$$f_{n+1,j}^{(i)} - f_{n,j}^{(i)} = -\alpha\left(f_{n,j}^{(i)} - f_{n,j-1}^{(i)}\right) + i\gamma\left(f_{n,j}^{(i+1)} + f_{n,j-1}^{(i+1)}\right) + i\beta\left|\left(f_{n,j}^{(i)} + f_{n,j-1}^{(i)}\right)\right|^2\left(f_{n,j}^{(i)} + f_{n,j-1}^{(i)}\right)$$

$$f_{n+1,j}^{(i+1)} - f_{n,j}^{(i+1)} = -\alpha\left(f_{n,j}^{(i+1)} - f_{n,j-1}^{(i+1)}\right) + i\gamma\left(f_{n,j}^{(i)} + f_{n,j-1}^{(i)}\right) + i\beta\left|\left(f_{n,j}^{(i+1)} + f_{n,j-1}^{(i+1)}\right)\right|^2\left(f_{n,j}^{(i+1)} + f_{n,j-1}^{(i+1)}\right)$$

$$\alpha = \frac{\Delta T}{n_g \Delta Z},\quad \gamma = \frac{\kappa \Delta T}{2 n_g},\quad \beta = \frac{n_2 \Delta T}{8 n_g}$$

$$f_{n,j}^{(i)} \equiv f^{(i)}(j\Delta Z, n\Delta T)$$



3. Accuracy and Stability

Alternatives of the scheme:

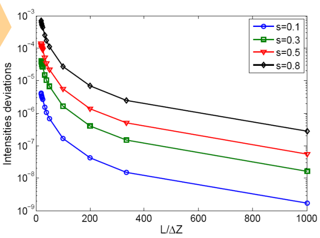
1) $i\gamma\left(f_{n,j}^{(i+1)} + f_{n,j-1}^{(i+1)}\right)$, $i\gamma\left(f_{n,j}^{(i)} + f_{n,j-1}^{(i)}\right)$

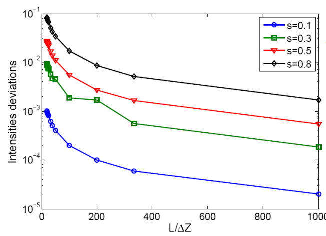
2) $i\gamma 2 f_{n,j}^{(i+1)}$, $i\gamma 2 f_{n,j-1}^{(i)}$

3) $i\gamma 2 f_{n,j-1}^{(i+1)}$, $i\gamma 2 f_{n,j}^{(i)}$

4) $i\gamma 2 f_{n,j}^{(i+1)}$, $i\gamma 2 f_{n,j}^{(i)}$

Deviations from analytical solution for linear coupler, L=10 μm

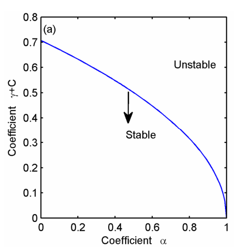


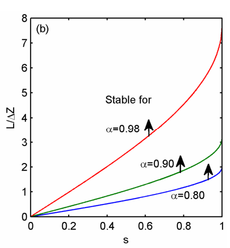


Stability criteria

(von Neumann stability analysis)

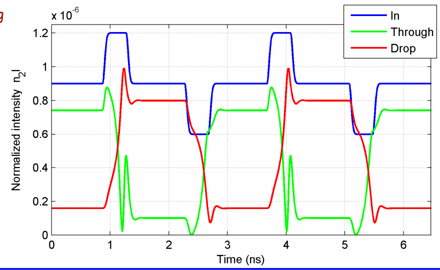
- $\alpha \le 1$
- $\left[\frac{\arcsin(s)}{L_C} + n_2 I_{\max}\right]\Delta Z \le \frac{\sqrt{2(1-\alpha)}}{\alpha}$





3. Numerical example: All-optical switching

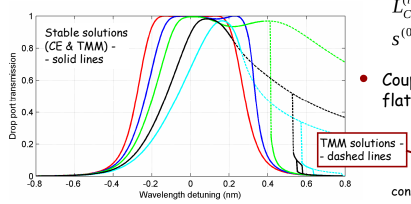
- bistable channel dropping filter: single ring structure with circumference $L = 31\pi$ μm
- resonance $\lambda_r \approx 1.5428$ μm
  $L_C^{(0)} = L_C^{(1)} = 4$ μm
  $s^{(0)} = s^{(1)} = 0.1$
  $n_g^{(i)} = 1.6$
- the input intensity is modulated by positive and negative pulses
- CE readily provides time dependencies



4. Numerical example: Self-pulsing

Drop port response near resonance

$\lambda_r \approx 1.5640$ μm



- Structure consists of 3 identical rings with circumferences $L = (7.0 + 13\pi)$ μm
  $L_C^{(i)} = 3.5$ μm $n_g^{(i)} = 4.25$
  $s^{(0)} = s^{(3)} = 0.4668;$ $s^{(1)} = s^{(2)} = 0.099$
- Coupling coefficients are optimized for flat response [1] => optical bistability reduced

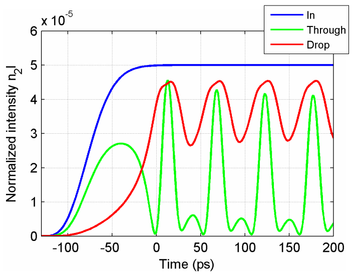
condition $dI_{\text{drop}} / dI_{\text{in}} > 0$ is valid.

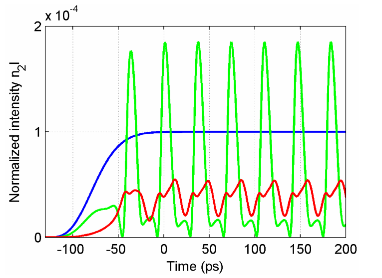
However, CE reveals that these solutions are unstable.

Unstable (self-pulsing) solutions

for wavelength detuning 0.5 nm

- amplitude and period of pulses depend on input intensity and detuning from resonance wavelength





Acknowledgements

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